

MATH PROBLEMS OF THE MONTH

December 2019 (Fall Series IV of IV)

1. **Arithmetic Mashup.** If you split the four-digit number 3025 with a plus sign in the middle, and square the result, it does a lovely trick: $(30 + 25)^2 = 3025$. Neat! And 2025 does, too: $(20 + 25)^2 = 2025$. Find a *ten*-digit number with the same property: If you put a plus sign in the middle, then square the sum, you get your ten-digit number back. (A heartbreaking near miss: $(11110 + 22222)^2 = 1111022224$, not quite the desired 1111022222.)

2. **All Right?** The graphs $y = \cos x$ and $y = \tan x$ cross each other infinitely many times. A good plot will show that, at each point of intersection, the graphs *appear* to meet at a 90-degree angle. Are all the intersections really exactly 90 degrees? Prove your answer.

3. **Group Up!** Suppose each student in a class of 27 writes down an arbitrary word using letters from the 26-letter English alphabet. It could be ALFTAN or SHE or GAUSS or I or BEPEDMDRT or SQUEEEEEEEEEEEEE or whatever. **Prove:** No matter what words were written, the instructor can find two groups of students – group A and group B – such that every letter of the alphabet used by anyone in group A is also used by at least one student in group B, and every letter used by anyone in group B is also used by at least one student in group A. Each group must include at least one student, and no student may belong to both groups. There may be some students who are not in either group.

Solutions are welcome from all Gustavus students, faculty, and staff! Each month's solvers will be announced along with a running scoreboard for the Fall Series. Prizes of \$125 (first place) and \$50 (runner up) will be awarded to the top student solvers at the end of the Fall Series; students who have solved at least three problems during the Fall Series are eligible for the prizes. To enter the contest:

- (1) Email solutions to jsiehler@gustavus.edu, or
- (2) Submit written solutions to Professor Siehler's mailbox (by the door of Olin Hall 310).

Please include your name and email address with written solutions. Points will be awarded for each correct, complete solution received by Friday, December, 20. Find the problems online at <https://mcs.blog.gustavus.edu/tag/potm/>.