

MATH PROBLEMS OF THE MONTH

October 2019 (Fall Series II of IV)

1. Change Four. Imagine that the following list of three 7-letter words is made from tiles that have X on one side, and O on the other side:

OOOXXXX, XXOXOOX, OXXXXOO.

Any one of the words can be changed to any of the others by flipping exactly four tiles (from X to O or vice versa). Construct a list of *eight* 7-letter words using Xs and Os, with the same property: any word in your list can be changed to any other word in your list by flipping exactly four tiles.

2. Let's Arrange A Transaction. Montana Gaines – you know, the famous game show host – offers you a deal: You may choose any real number a . Once you choose, she'll write down the function $f(x) = x^2 - 2ax - 40x + 60a + 400$, using your chosen value for a . Then she will pay you the *absolute minimum value* of this function, in dollars (and if that minimum value is negative, you owe *her* money). What value of a should you choose to maximize your payout, and what payout do you get?

3. Best Factor Friends. For a positive integer n , let $P(n)$ denote the set of prime numbers which go evenly into either n or $n + 1$. If a and b are distinct positive integers with $P(a) = P(b)$, then we say that $a \heartsuit b$. For example, $11 \heartsuit 242$ since $P(11) = P(242) = \{2, 3, 11\}$. Show that there are infinitely many pairs of positive integers a and b with $a \heartsuit b$. (I've already given you one pair, which is a good start; now just find infinitely many more.)

Solutions are welcome from all Gustavus students, faculty, and staff! Each month's solvers will be announced along with a running scoreboard for the Fall Series. Prizes of \$125 (first place) and \$50 (runner up) will be awarded to the top student solvers at the end of the Fall Series; students who have solved at least three problems during the Fall Series are eligible for the prizes. To enter the contest:

- (1) Email solutions to jsiehler@gustavus.edu, or
- (2) Submit written solutions to Professor Siehler's mailbox (by the door of Olin Hall 310).

Please include your name and email address with written solutions. Points will be awarded for each correct, complete solution received by Sunday, October 27. Find the problems online at <https://mcs.blog.gustavus.edu/tag/potm/>.