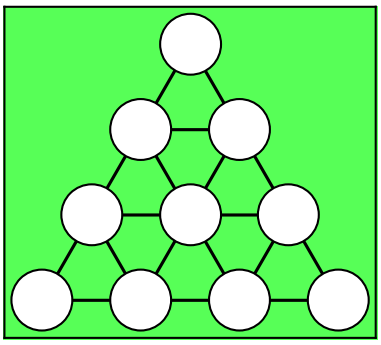
MATH PROBLEMS OF THE MONTH

September 2019 (Fall Series I of IV)

I. The sociology of numbers



1. Numbers which differ by less than three (like 4 and 6, or 10 and 11) are bitter enemies who can't stand to be near each other. Numbers which differ by three or more are always good friends and rejoice in each other's company. Fill in the circles with the numbers 1–10 so that every number is connected only to friends, never to enemies. Use each number once.

2. How many different solutions are possible for problem #1? State the number of solutions clearly, and prove that your answer is correct.

II. The limit *does* exist

3. Let f_n denote the n-th number in the Fibonacci sequence $(0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots,$ and so on). Let A_n denote the area of the triangle with vertices $(0, 0)$, (f_n, f_{n+1}) and (f_{n+1}, f_{n+2}). Evaluate $\lim_{n\to\infty} A_n$.

Solutions are welcome from all Gustavus students, faculty, and staff! Each month's solvers will be announced along with a running scoreboard for the Fall Series. Prizes of $125 (first place) and $50 (runner up) will be awarded to the top student solvers at the end of the Fall Series; students who have solved at least three problems during the Fall Series are eligible for the prizes. To enter the contest:

(1) Email solutions to jsiehler@gustavus.edu, or
(2) Submit written solutions to Professor Siehler's mailbox (by the door of Olin Hall 310).

Please include your name and email address with written solutions. Points will be awarded for each correct, complete solution received by Friday, September 27. Find the problems online at https://mcs.blog.gustavus.edu/tag/potm/.