MATH PROBLEMS OF THE MONTH

December 2018 (Fall Series IV of IV)

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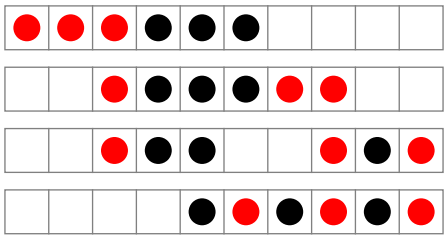
1. Each move in the game below consists of sliding two checkers from adjacent boxes to two open positions, keeping them adjacent and in their original order as they move. In three moves, the pattern is changed from sorted colors to alternating colors:



In four moves, change these eight checkers to alternating colors, with no gaps between them:

2. Suppose a, b, c, and d are integers such that $f(x) = ax^3 + bx^2 + cx + d$ has three irrational roots. What is the smallest possible value of $|f(1/7)|$?

3. An infinitely differentiable function F satisfies the identity

$$F(x) = (1 + x + x^2)F(x^2)$$

for every x in the interval $(-1, 1)$, and $F(0) = 1$. Find the value of $F^{(10)}(0)$, that is, the 10th derivative of F at the origin.

Solutions are welcome from all Gustavus students, faculty, and staff! Each month's solvers will be announced along with a running scoreboard for the Fall Series. Prizes of $125 (first place) and $50 (runner up) will be awarded to the top student solvers at the end of the Fall Series; students who have solved at least three problems during the Fall Series are eligible for the prizes. To enter the contest:

(1) Email solutions to jsiehler@gustavus.edu, or
(2) Submit written solutions to Professor Siehler's mailbox (by the door of Olin Hall 310).

Please include your name and email address with written solutions. Points will be awarded for each correct, complete solution received by Thursday, December 20th. Find the problems online at https://mcs.blog.gustavus.edu/tag/potm/.