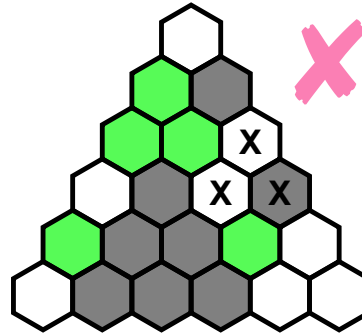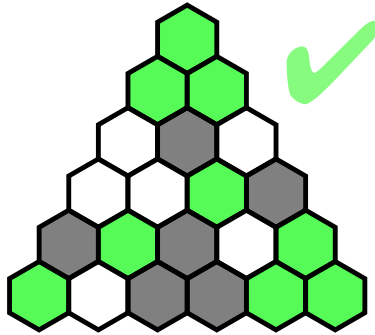# MATH PROBLEMS OF THE MONTH

October 2018 (Fall Series II of IV)

I. Coloring!   A pyramid of hexagonal cells is to be colored green, white, and/or gray. There's a rule, though: Each cell taken together with the two cells directly below it must be either (1) all three the same color, or (2) all three different colors. The coloring on the left is correct. The one on the right is *nearly* correct, but has a problem at the marked cells.

1. How many ways are there to color the six-row pyramid correctly?

2. A pyramid with 30 rows is colored according to the same rule. There is at least one green cell. What is the maximum number of cells that could be gray? (State your answer clearly, and prove that it really is the maximum.)

II. Intersectionality.

3. The endpoints of three intervals (a_1, b_1), (a_2, b_2), (a_3, b_3) are chosen randomly with a uniform distribution in the interval $[0, 1]$. What is the probability that at least one of the intervals intersects both of the other intervals? (Assume that the endpoints of each interval have put in order, $a_1 < b_1$ and so on, so that all three intervals are nonempty.)

Solutions are welcome from all Gustavus students, faculty, and staff! Each month's solvers will be announced along with a running scoreboard for the Fall Series. Prizes of \$125 (first place) and \$50 (runner up) will be awarded to the top student solvers at the end of the Fall Series; students who have solved at least three problems during the Fall Series are eligible for the prizes. To enter the contest:

    (1) Email solutions to jsiehler@gustavus.edu, or
    (2) Submit written solutions to Professor Siehler's mailbox (by the door of Olin Hall 310).

Please include your name and email address with written solutions. Points will be awarded for each correct, complete solution received during the month of October, 2018. Find the problems online at https://mcs.blog.gustavus.edu/tag/potm/.